

# CORRECTED NOTES ON NEGATIVE NUMBERS

FOR PROJECT SHIVOHAM

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## CHINA BEAT INDIA BY CENTURIES TO A CORRECT ARITHMETICAL TREATMENT OF POSITIVE AND NEGATIVE NUMBERS

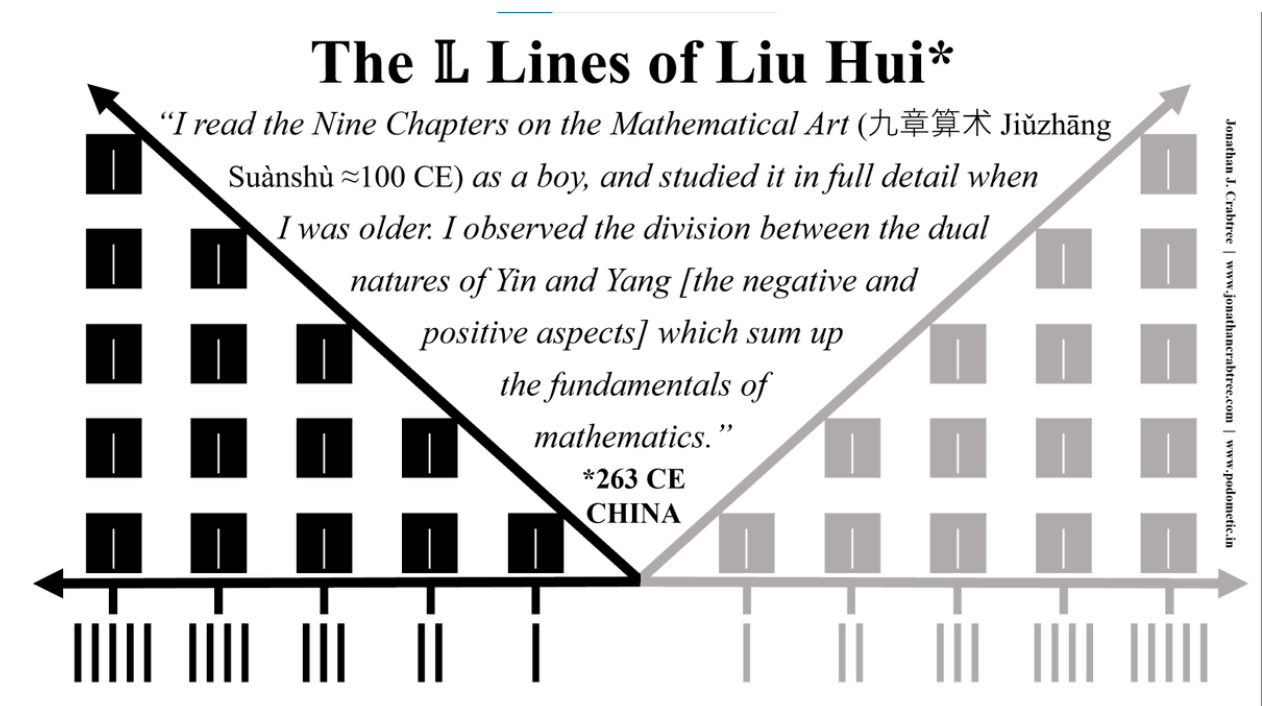
### Yin and Yang (Negative and Positive)

China was comfortable with positive and negative numbers around 2000 years before the West. To the Chinese, the idea of Yin and Yang, or opposing yet interconnected forces, fits beautifully with arithmetic.

In 263 CE, Liu Hui wrote a commentary on the ancient *The Nine Chapters on the Mathematical Art*, (九章算术 Jiǔzhāng Suànshù circa 100 CE). Liu Hui said:

*I read the Nine Chapters as a boy, and studied it in full detail when I was older. I observed the division between the dual natures of Yin and Yang [the positive and negative aspects] which sum up the fundamentals of mathematics.*

SOURCE *The Nine Chapters on the Mathematical Art: Companion and Commentary*, Shen Kangshen, John N. Crossley and Anthony W. C. Lun, Oxford University Press, 2000.

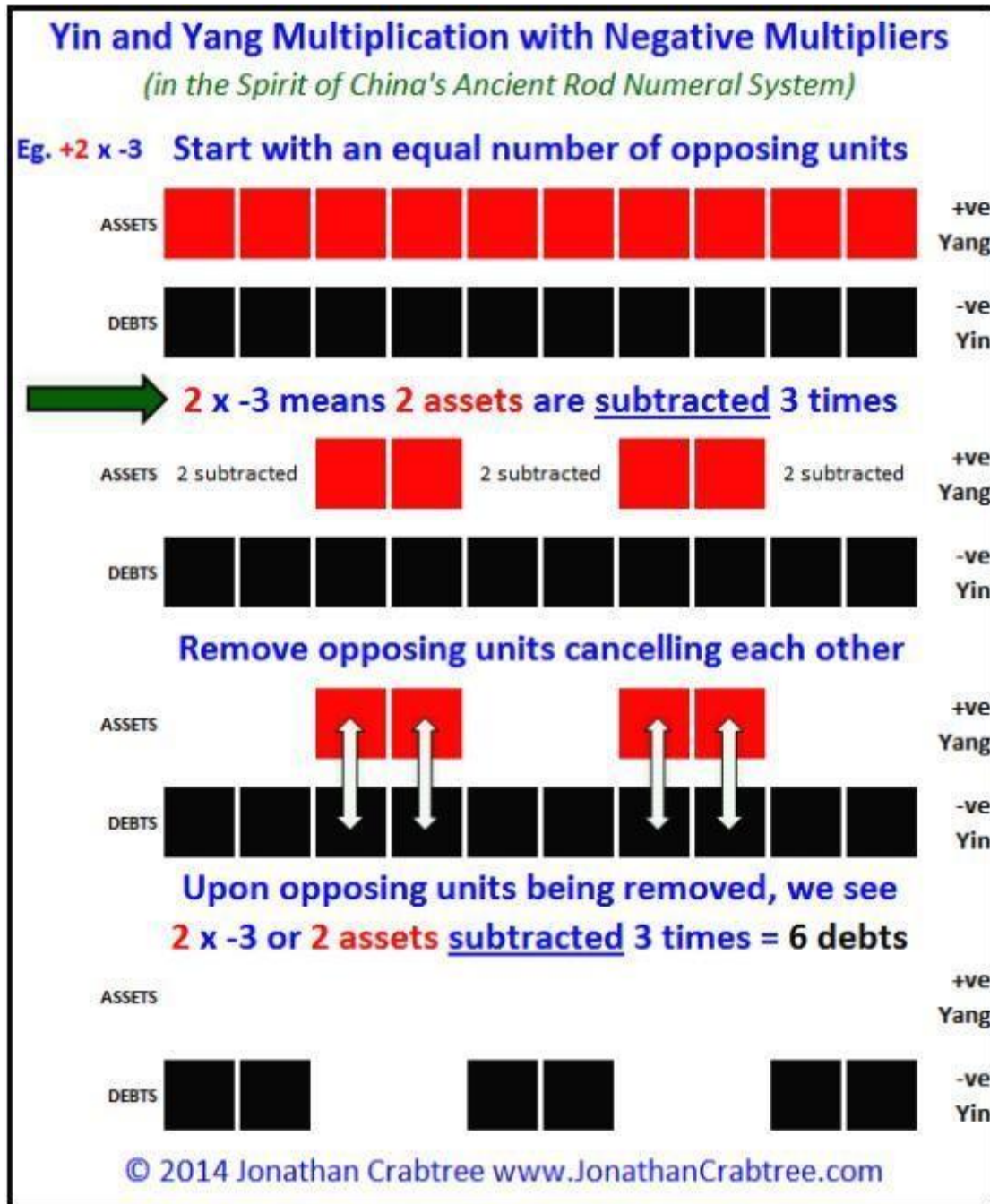


From the Sanskrit of Brahmagupta, in 628 CE zero was defined to be the sum of a positive number and negative number of equal magnitude, (Brāhma Sphuta-siddhānta, Chapter 18:30a).

To me, a secret, seemingly lost, is that Yin and Yang can embed the concept of positive and negative into the unit. We may not need negative numbers at all when two opposing and cancelling units exist. Let me explain with the typical example of debts being equated with 'negative' and assets beings equated with 'positive', which was common in both China and India centuries before such an idea was considered in the West.

***Yin & Yang Multiplication of 2 things (assets) multiplied by -3.***

*(Please note, Ancient China used red for positive and black for negative, which is the reverse of modern accounting practices.)*



Examples such as this, based on Yin and Yang principles, are consistent with how **integer tiles** can be used in classrooms to reveal integer arithmetic. Now I will introduce the idea of primary units of count or measurement with secondary units of count or measurement that cancel each other out. For example, **Population** can be a **Primary Unit**. Assume the population of a town is 100. We can count births (positive) and we can count deaths (negative). The opposing, yet interconnected **Births and Deaths** are **Secondary Units**, that together, comprise the **Primary Unit of Population**.

Assume every year there are 10 births and 10 deaths. The number of 10 births equals the number of 10 deaths and so there is ZERO net change in our Primary Unit.

There are 20 events a year, yet consistent with the opposing, interconnected roles of life and death, the opposing forces (births and deaths) cancel one another out and both births and deaths are counted with 'natural' or 'counting' numbers.

So what happens when we take away the 'positive' yang events of birth?

Population start 100 + zero births + 10 deaths = population end 90. Change = 10 less.

And what happens when we take away the 'negative' yin events of death?

Population start 100 + 10 births + zero deaths = population end 110. Change = 10 more.

Taking away deaths, with no change in births results in an increase in population.

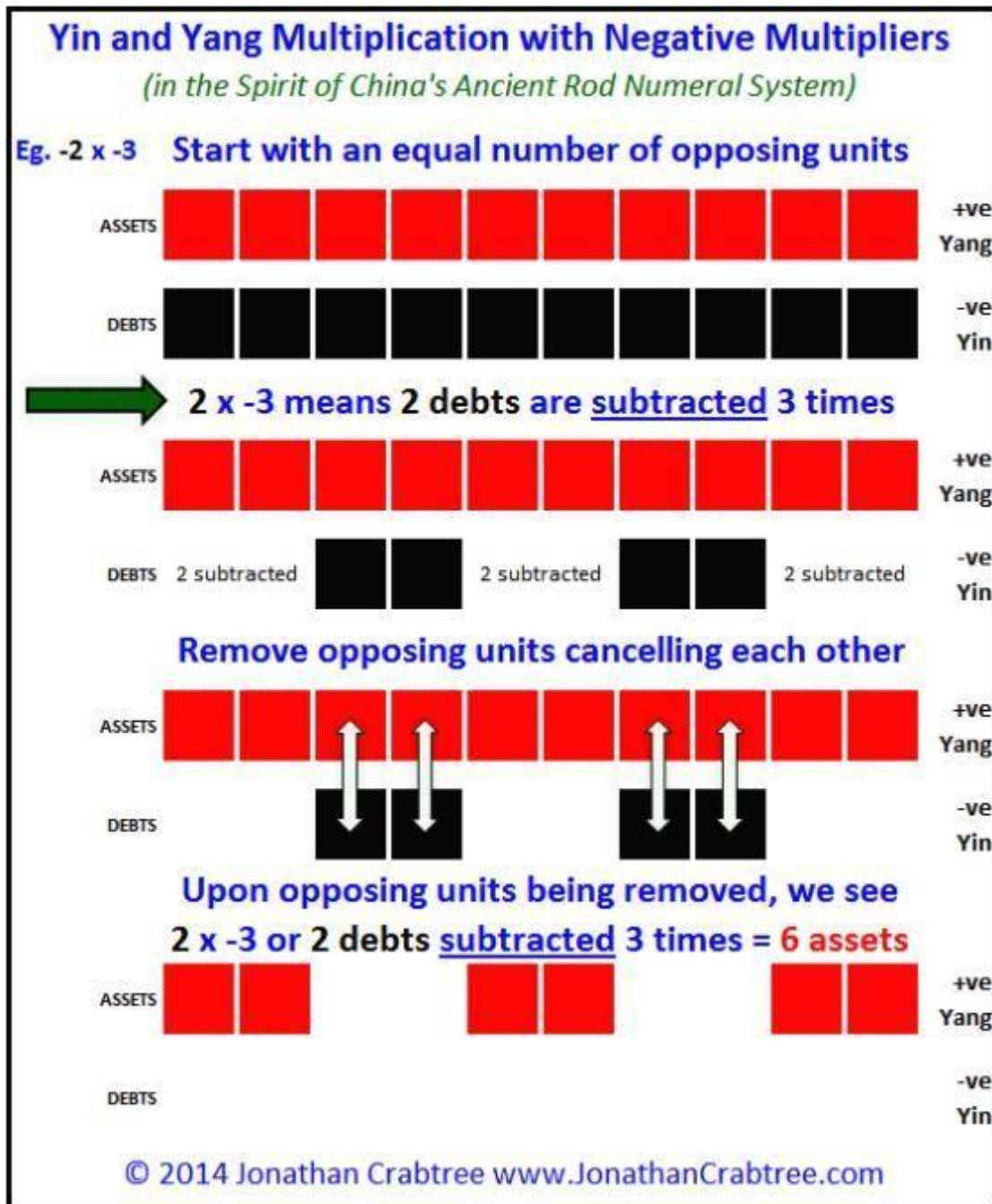
Even the words 'less' and 'more' are Yin and Yang Secondary Units in essence, as they are contrary yet interconnected ideas that can cancel each other out. Again, the words 'more' and 'less' are concepts in which the sign (positive or negative) is embedded into the Primary Unit we might call Change.

The Chinese Rod Numeral System used the opposing colors of red and black to represent ideas such as assets and debts. When a count of two units of debt (a black two) and a count of two units of wealth (a red two) appeared on the counting board, they would both be removed as they offset each other.

If a Chinese man had 4 assets and then had 6 assets taken away the red 4 on the counting board would be removed and a black two would appear to count two debts. The opposing yet interconnected concepts of assets and debts would arithmetical foundations in both China and India. Thus there could be no numbers less than zero in China because upon crossing from one side of the counting board to the other, you

simply changed the unit of count, from asset to debt, or debt to asset. Zero was not needed in China and numbers when arranged in columns to show place value, would simply have empty places.

*So let's look at Yin & Yang Multiplication of 2 things (debts) multiplied by -3.*



Instantiations of China's dual YinYang approach to arithmetic reveal how integer multiplication really works. Negative two (2 debts) multiplied by negative three (subtracted from zero three times) equals positive six or six assets! So in  $-2 \times -3$  the first - symbol indicates the Yin Yang concept of negative or positive and the second -

symbol simply reveals the operation of subtraction, with the integral multiplier saying how many times the positive or negative (assets or debt) is subtracted from zero. Why zero? Well China didn't have zero, yet they did have the idea of cancelling opposite units. So starting with ten positives and ten negatives is a way to start with India's idea of zero.

### *Yin and Yang in the Bathtub!*

Integer tiles depicting opposing Yin Yang units can be fun for children to play with. Assume the temperature of the water in your bathtub is just right. If you add a cup of hot water, the bathwater heats up. If you add a cup of cold water, the bathwater cools down. In Australia, hot taps often have a red icon while cold taps have a blue icon.

So instead of red and black 'integer tiles' being opposing colors, you might choose to use red and blue instead with drawings of a bathtub (for safety!)

Assume the bathtub temperature in your drawing is body temperature and just right for your child without being too hot or too cold.

Add 2 hot blocks (red) and the temperature goes up 2.

Add 2 cold blocks (blue) and the temperature goes down 2.

Add 2 hot blocks and 2 cold blocks and the temperature stays the same.

Now assume your body temperature bathtub starts with 10 hot blocks and 10 cold blocks.

**There are two ways to increase the temperature of your bath.**

- Add hot blocks or subtract cold blocks; the result is the same!

**There are two ways to decrease the temperature of your bath.**

- Add cold blocks or subtract hot blocks; the result is the same!

Children will quickly understand what happens to the bathwater when you subtract cold blocks. Temperature goes up! If you assign positive to heat and negative to cold, then subtracting cold results in more heat, which is a positive change.



China's Yin Yang secret is arithmetic becomes much simpler when we play with opposing units that cancel each other out. On many occasions, there need not be numbers 'less than zero' because we can switch to an inverse or opposing unit!

Incorporating the sign (positive or negative) into the units instantly makes confusing arithmetic simple.

***Do this experiment for yourself!***

Ask a random person (or non-mathematician) the following question.

***"What is negative seven minus negative four?"***

My experience is, people usually respond with "I was never good at math" or "I don't have a head for numbers". Say that's ok, and you're just curious and interested to know.

Many if not most people will answer with a tentative "*Negative eleven?*" Instead of saying "**Wrong**", just apply China's Yin Yang secret and rephrase the question.

***"If you have seven negatives and four negatives are taken away, how many remain?"***

Almost every time, the answer will come back "**Three negatives**".

China converted arithmetical adjectives into arithmetical nouns. The word negative in "negative seven" is an adjective and the word seven is a noun in the absence of a unit.

The secret is to make the word 'seven' an adjectival count and the word 'negative' a noun! So of course 7 negatives minus 4 negatives equals three negatives! That's just as simple as negative eleven minus negative four is confusing!

When it comes to multiplication, it is important to know the correct (new) definition of integer multiplication, which involves either **repeated subtraction** of the number multiplied from zero or **repeated addition** of the number multiplied from zero, according to the sign of the integral multiplier.

In the case of China's Rod Numeral System, simply start your calculation with equal numbers of opposing units as a proxy for zero, then either add or subtract the relevant

Yin Yang or negative/positive unit as required, then remove the remaining units that cancel each other out, and there, before your eyes, is the answer!

*Nouns are much easier to play with than adjectives!*

Often we deal with a **Primary Unit** which has two opposing **Secondary Units**. So we no longer need numbers less than zero. Passing from either side of zero to the other side simply means the secondary unit of count or measure inverts!

A negative number is just the result of subtracting a larger number from a smaller number.

- The negative number for assets can then be counted with debts.
- The negative number for Easterly can be counted with Westerly.
- The negative number for more can be counted with less.

So let's look at a Yin Yang dual approach to integer multiplication with opposing Secondary Units.

*Subtracting Yin (negative) adds Yang (positive)*

*...and subtracting Yang, or positive, adds Yin, or negative.*

**META CONCEPTS AND THEIR YIN YANG OPPOSING ELEMENTS**

HORIZONTAL DIRECTION North/South, Left/Right

VERTICAL DIRECTION East/West, Up/Down

FINANCIAL POSITION Assets/Debts, Profit/Loss

POPULATION Births/Deaths, Immigration/Emigration

TEMPERATURE Hot/Cold, Above Zero/Below Zero (Centigrade)

SUFFICIENCY More Than Enough/Less Than Enough

SEA LEVEL Above/Below

...are all dual concepts that require their interconnected 'inverse' in order to have meaning.

Taking away debt results in an increase in wealth.

*1 Yang asset + 1 Yin debt = zero net financial position*

*1 Yang asset = 1 Yang asset*

*Taking away an asset results in an increase in debt.*



*1 Yin debt + 1 Yang asset = zero net financial position*

*1 Yin debt = 1 Yin debt*

Taking away cold results in an increase in temperature.

*1 Yang heat + 1 Yin cold = zero net change in temperature*

*1 Yang heat = 1 Yang heat*

So the opposing forces of Yin and Yang are part of our universe. Zero can be considered the sum of all numbers on our bi-directional number line. Just as multiplication can involve repeated addition from zero, it can also involve repeated subtraction from zero. (YinYang)

So if the idea of negative numbers has gone missing in the West, (it has) then maybe that can explain why half our arithmetic is missing!

The identity elements are one and zero. Any number multiplied by one, or divided by one, keeps its identity and remains unchanged. Any number that has zero added to it, or zero subtracted from it, keeps its identity and remains unchanged.

**Returning the identity elements to arithmetic reveals patterns not often seen.**

We'll start with simple integral multiplication.

$$a \times (+4) = 0 + a + a + a + a$$

$$a \times (+3) = 0 + a + a + a$$

$$a \times (+2) = 0 + a + a$$

$$a \times (+1) = 0 + a$$

$$a \times (0) = 0$$

The pattern, row by row, is as each integral multiplier reduces by one, we subtract an 'a'. So let's continue the pattern of integral multipliers reducing by one, line by line.

$$a \times (+4) = 0 + a + a + a + a$$

$$a \times (+3) = 0 + a + a + a$$

$$a \times (+2) = 0 + a + a$$

$$a \times (+1) = 0 + a$$

$$a \times (0) = 0$$

$$a \times (-1) = 0 - a$$

$$a \times (-2) = 0 - a - a$$

$$a \times (-3) = 0 - a - a - a$$

$$-a - a - a \times (-4) = 0 - a$$

$$-a - a - a$$

So integral multiplication involves either repeated addition or repeated subtraction, depending on the sign of the multiplier.

OK, we reintroduced the identity element *zero* into the pattern of multiplication, so what if we reintroduced the identity element *one* into the pattern of exponentiation? This is what Gauss did and he was rather good at maths.

$$a^{(+4)} = 1 \times a \times a \times a \times a$$

$$a^{(+3)} = 1 \times a \times a \times a$$

$$a^{(+2)} = 1 \times a \times a \ a^{(+1)}$$

$$= 1 \times a \ a^{(0)} = 1$$

The pattern, row by row, is as each integral exponent reduces by one, we divide by 'a'. So let's continue the pattern of integral exponents reducing by one, line by line.

$$a^{(+4)} = 1 \times a \times a \times a \times a$$

$$a^{(+3)} = 1 \times a \times a \times a$$

$$a^{(+2)} = 1 \times a \times a \ a^{(+1)}$$

$$= 1 \times a \ a^{(0)} = 1 \ a^{(-1)} =$$

$$1 \div a \ a^{(-2)} = 1 \div a \div a$$

$$a^{(-3)} = 1 \div a \div a \div a \ a^{(-$$

$$4) = 1 \div a \div a \div a \div a$$

The English language definition ***ab = a added to itself b times*** quoted by mathematicians for centuries is misleading because of an incorrect translation in 1570 of Euclid's definition of multiplication, that dates back to 300 BCE.

The Collins dictionary says an exponent is *"a number or variable placed as a superscript to the right of another number or quantity indicating the number of times the number or quantity is to be multiplied by itself."*

**Source:** [www.collinsdictionary.com/dictionary/english/exponent](http://www.collinsdictionary.com/dictionary/english/exponent)

*The English language definition of exponentiation  $a^b$*

*= a multiplied by itself b times is also wrong*

In  $a^b$ , when  $b = 1$ , we are told that  $a^1$  equals a multiplied by itself one time, or  $a \times a$ . However,  $a \times a$  is 'a squared' or 'a to the power of two', or ' $a^2$ '.

An exponentiation definition you won't see is  $a^b =$  one multiplied by  $a$ ,  $b$  times in a row.

*Exponentiation such as  $a^{(+3)}$  is  $1 \times a \times a \times a$  and exponentiation like  $a^{(-3)}$  is  $1 \div a \div a \div a$ , so integral exponentiation is either repeated multiplication or repeated division, as per the sign of the exponent!*

Decay and Growth are opposing natural and interconnected concepts that may be interpreted in the manner in Yin and Yang. The mathematical concept that matches decay and growth is exponentiation. When the exponent is positive (Yang), repeated multiplication is involved and when the exponent is negative (Yin), repeated division is involved.

To quote again from G H Hardy's "A Mathematician's Apology":

*I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations', are simply our **notes of our observations**.*

Yes, multiplication involves repeated subtraction as well as repeated addition and exponentiation involves repeated division as well as repeated multiplication.

Examples of authors discussing math as a *science of patterns* include: *Mathematics as a science of patterns*, by Michael D Resnik; *Mathematics, the science of patterns*, by Keith Devlin and *Mathematics as the Science of Patterns*, by Michael N. Fried. Patterns are important, yet so too are structure and relationships. I didn't like mathematics as a child because sometimes the patterns I saw didn't match what I was being taught by my teachers. So these articles are *notes of my observations*. Exponentiation involves division, just as exponentiation involves multiplication. Multiplication involves subtraction, just as multiplication involves addition.

Yin and Yang. Go figure...